MATH 180 FINAL EXAM PRACTICE

1. Determine the **limit**: $\lim_{x \to 2} \frac{2x^2 + 4x - 16}{x^2 - 4}$

2. Evaluate the following **limit**: $\lim_{x \to 0} \frac{e^{2x} - 1}{x}$

3. Suppose that for some function $f(x)$, we know that $\lim_{x \to 2^-} f(x) = 3$ and $\lim_{x \to 2^+} f(x) = 3$. We also know that $f(2) = 2$. Does the limit as x approaches 2 exist? If so, what is it? Is the function continuous at x = 2? If not, what type of discontinuity is experienced?

4. Below is a table that gives the distances a car has traveled (in feet) at various times (in seconds). Use the information to **estimate the instantaneous velocity** at t = 3.

Time, t	0	1	2	3	4	5	6
Distance, d	0	10	32	70	119	178	239

5. The equation of the tangent line to some function at x = 1 is known to be 4x+2y=6. What was the outcome of the following limit: $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for that particular function (at x = 1)?

6. Find the exact values of x at which **horizontal tangents** exist for the function: $g(x) = x(5x-6)^3$

7. Find the **velocity at t = 3** for the distance function $d(t) = \frac{2t^2}{\sqrt{t+1}}$

8. Given $f(x) = 9x^3 + 5\ln 3x$, write the **equation of the line that is tangent** to the function at $(\frac{1}{3}, \frac{1}{3})$.

9. Given the function $f(x) = \begin{cases} (x-4)^2 - 5 & \text{if } x \le 0 \\ 2x - 8 & x > 0 \end{cases}$, determine the **limit** $\lim_{x \to 5} f(x)$ if it exists.

9b. What is the **slope of the function** at x = -1?

10. Given the function $f(x) = 2x^2 e^{2x}$, find the **instantaneous rate of change** when x = 0.

11. At **what values of x** does the curve $y = x^3 + 2x$ have a tangent line parallel to the line y = 14x + 10?

12. Given $f(x) = \sin^2(2x)$, what is the value of $f'(\frac{\pi}{6})$?

13. A 12 foot long ladder leans against a wall. If the bottom of the ladder slides away from the wall at a rate of 1 foot per second, **how fast is the top of the ladder sliding down the wall** when the bottom of the ladder is 3 feet away from the wall?

14. Find the **absolute maximum and absolute minimum** value of the function $f(x) = x\ln 2x$ on the interval [0.10, 6]

15. Find the **interval(s) over which the function** $f(x) = x - 2\cos x$ **is increasing**, with the restriction that $0 \le x \le 2\pi$.

16. For the following problem, consider the function $f(x) = 2x^3 - 3x^2 - 12x + 1$

A. Find the **intervals of x over which the function is increasing and decreasing.**

B. Find all points at which **horizontal tangent lines exist.**

C. Now find the **second derivative** and use it to find the x value at which concavity changes.

D. Using the information in parts A through C and the end behavior of the graph, sketch the graph

17. Given the function $f(x) = \frac{1}{4}x^4 - 6x^2 + 1$, determine the **intervals of concavity and inflection points**

18. The volume function for a box is $V = (12 - 2x)^2 x$ Find the x value that would **maximize the volume** of the box. Then use the 2nd derivative test to make sure it's maximized at that x value (rather than minimized).

19. Find the **area under the curve** $f(x) = x^2\sqrt{1 + x^3}$ from x = 0 to x = 2

23. What is the **value of the integral** $\int_0^{\frac{\pi}{4}} (\sin^2 x \cos x)dx$

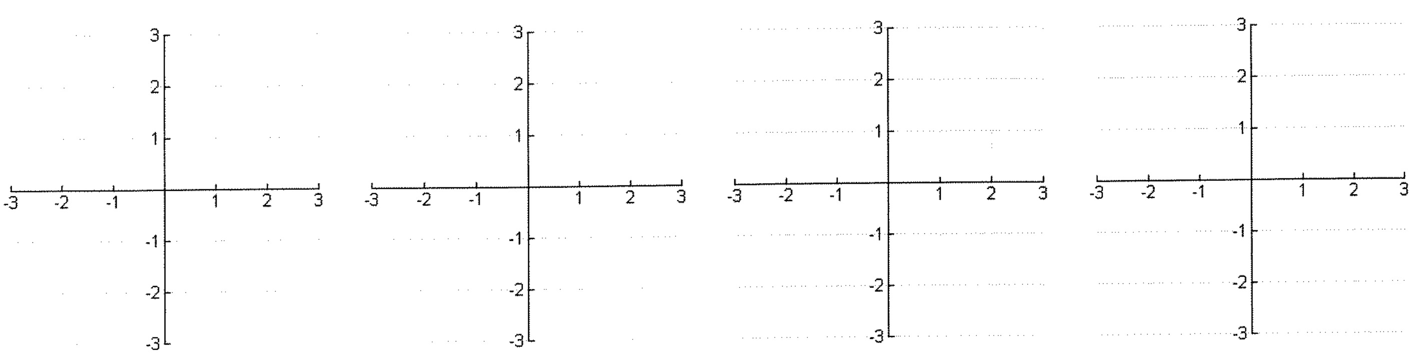
FOR PROBLEMS 21-24, USE THE GRAPHS BELOW.

21. Find the **total area enclosed** by the curves $y = x$ and $y = \sqrt[3]{x}$

22. **Use the method of cylindrical shells** to determine the volume of the solid obtained by rotating the region bounded by the curves $y = (x-1)^2$, y = 0, x = 0, and x = 1 about the line x = 1

23. **Use the disk/washer method** to find the volume of the solid formed by the curves $x = 1 - y^4$ and x = 0, and rotated about the y-axis.

24. Use whichever method seems easier to **evaluate the volume of the solid** obtained when rotating the region bounded by the following curves about the y-axis. $y = e^{-x^2}$, y = 0, x = 0, and x = 1.



25. Find the **average value of the function** $f(x) = \frac{5}{\sqrt{x}}$ on the interval [1,4]

26. Find the **arclength** of the function $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ from x = ½ to x = 1

27. Find the **surface area** of the object obtained by rotating $y = (x-1)^2$ about the x-axis. Let x be on [-1,1]

1) 3

2) 2

3) yes 3 no removable

4) 4 3 5

5) −2

6) $\frac{8}{5}$, $\frac{2}{10}$

7) $\frac{39}{8}$

8) $y = 18x - 17/3$

9) 2 , −10

10) $\frac{\pi a}{0}$

11) ±2

12) $\sqrt{3}$

13) −.258

14) −.184 min
14.91 max

15) $(0, \frac{7\pi}{6})$ $(\frac{11\pi}{6}, 2\pi)$

16) Dec $(-1, 2)$
Inc $(-\infty, 1)$ $(2, \infty)$

B) $(-1, 8)$ $(2, -19)$

C) 1/2

17) CD $(-2, 2)$
CV $(-\infty, -2)$ $(2, \infty)$
inf $(-2, -19)$ $(2, -19)$

18) 2

19) 52/9

20) $\frac{\sqrt{2}}{12}$

21) 1/2

22) $\frac{\pi}{2}$

23) $\frac{64\pi}{45}$

24) $-\pi\left(\frac{1}{e} - 1\right)$

25) 10/3

26) 31/48

27) $\pi\left(\frac{17\sqrt{17} - 1}{6}\right)$